

$$1) \quad 1 = \int_{-\infty}^{+\infty} \psi^* \psi dx = |c|^2 \int_0^{\infty} e^{-2\gamma x} dx = \frac{|c|^2}{2\gamma} \Rightarrow c = \sqrt{2\gamma} e^{i\alpha}$$

where  $\alpha$  can take any real value. For simplicity we will assume  $\alpha = 0$ .

2) First let us list the following two integrals:

$$\int_0^{\infty} x e^{-\beta x} dx = \frac{1}{\beta^2} \quad \int_0^{\infty} x^2 e^{-\beta x} dx = \frac{2}{\beta^3}$$

Now

$$\langle x \rangle = |c|^2 \int_0^{\infty} x e^{-2\gamma x} dx = 2\gamma \cdot \frac{1}{(2\gamma)^2} = \frac{1}{2\gamma}$$

$$\langle x^2 \rangle = |c|^2 \int_0^{\infty} x^2 e^{-2\gamma x} dx = 2\gamma \cdot \frac{2}{(2\gamma)^3} = \frac{2}{(2\gamma)^2}$$

$$3) \quad \Delta = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2}{(2\gamma)^2} - \frac{1}{(2\gamma)^2}} = \frac{1}{2\gamma}$$

4) Probability of finding the particle in  $[-2, 0]$  is zero as the wave function is zero in this range

$$5) \quad P(0 \leq x \leq 6) = \int_0^6 |\psi|^2 dx = |c|^2 \int_0^6 e^{-2\gamma x} dx =$$

$$= 2\gamma \cdot \frac{1}{2\gamma} (1 - e^{-2\gamma \cdot 6}) = 1 - e^{-1} \approx 0.63$$