Inserting ψ into the stationary Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

and using the fact that

$$\psi' = (-4\alpha x^3)\, C e^{-\alpha x^4} \qquad \psi'' = (-12\alpha x^2 + 16\alpha^2 x^6)\, C e^{-\alpha x^4}$$

we get

$$\left[-\frac{\hbar^2}{2m}(-12\alpha x^2 + 16\alpha^2 x^6) + V(x) - E\right] C e^{-\alpha x^4} = 0$$

Since E is a constant

$$V(x) = \frac{\hbar^2}{2m}(16\alpha^2 x^6 - 12\alpha x^2) + \beta$$

$$= \frac{\hbar^2}{m}(8\alpha^2 x^6 - 6\alpha x^2) + \beta$$

$$E = 0 + \beta$$

where β is an arbitrary constant. The potential in the Schrödinger equation can always be shifted by a constant without affecting anything.