1. No, Ψ is not a stationary state. While ϕ_0 and ϕ_1 are solutions of the stationary Schrödinger equation, Ψ is not. Ψ is a mixture of two eigenstates that correspond to different energies. Thus, each term in the linear combination will evolve differently and the probability density, $|\Psi(x,t)|^2$ will be changing with time.

2.
$$1 = \int |\psi|^2 = |c|^2 \int \left[2\phi_0^* + i\phi_1^*\right]\left[2\phi_0 - i\phi_1\right] dx =$$
$$= |c|^2 \Big(4 \underbrace{\int |\phi_0|^2 dx}_{1} + \underbrace{\int |\phi_1|^2 dx}_{1} + 2i \underbrace{\int \phi_1^* \phi_0 \, dx}_{0} - 2i \underbrace{\int \phi_0^* \phi_1 \, dx}_{0} \Big) =$$
$$= 5|c|^2 \implies c = \frac{1}{\sqrt{5}}$$

3.
$$\Psi(x,t) = \frac{2}{\sqrt{5}} \phi_0(x) e^{-\frac{iE_0 t}{\hbar}} - \frac{i}{\sqrt{5}} \phi_1(x) e^{-\frac{iE_1 t}{\hbar}} =$$
$$= \frac{2}{\sqrt{5}} \phi_0(x) e^{-i\frac{\omega}{2}t} - \frac{i}{\sqrt{5}} \phi_1(x) e^{-i\frac{3\omega}{2}t}$$
$$|\Psi(x,t)|^2 = \left(\frac{2}{\sqrt{5}} \phi_0^* e^{i\frac{\omega}{2}t} + \frac{i}{\sqrt{5}} \phi_1^* e^{i\frac{3\omega}{2}t}\right)\left(\frac{2}{\sqrt{5}} \phi_0 e^{-i\frac{\omega}{2}t} - \frac{i}{\sqrt{5}} \phi_1 e^{-i\frac{3\omega}{2}t}\right) =$$
$$= \frac{4}{5}|\phi_0|^2 + \frac{1}{5}|\phi_1|^2 - \frac{2i}{5}\phi_0\phi_1 e^{-i\omega t} + \frac{2i}{5}\phi_1\phi_0 e^{i\omega t} =$$
$$= \frac{4}{5}|\phi_0|^2 + \frac{1}{5}|\phi_1|^2 - \frac{4}{5}\phi_0\phi_1 \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \frac{4}{5}|\phi_0|^2 + \frac{1}{5}|\phi_1|^2 - \frac{4}{5}\phi_0\phi_1 \sin\omega t$$

4. Yes, $\Psi(x,t)$ is a periodic function with a period of $T = \frac{2\pi}{\omega}$ (after we take out the common factor $e^{-i\frac{\omega}{2}t}$). All expectation values will oscillate with this period.

5.

$$\langle H \rangle = \int \left[\frac{2}{\sqrt{5}}\Phi_0 e^{i\frac{\omega}{2}t} + \frac{i}{\sqrt{5}}\Phi_1 e^{i\frac{3\omega}{2}t}\right] H \left[\frac{2}{\sqrt{5}}\Phi_0 e^{-i\frac{\omega}{2}t} - \frac{i}{\sqrt{5}}\Phi_1 e^{-i\frac{3\omega}{2}t}\right] dx =$$

$$= \int \left[\frac{2}{\sqrt{5}}\Phi_0 e^{i\frac{\omega}{2}t} + \frac{i}{\sqrt{5}}\Phi_1 e^{i\frac{3\omega}{2}t}\right]\left[E_0 \frac{2}{\sqrt{5}}\Phi_0 e^{-i\frac{\omega}{2}t} - E_1 \frac{i}{\sqrt{5}}\Phi_1 e^{-i\frac{3\omega}{2}t}\right] dx =$$

$$= \frac{4}{5}E_0 + \frac{1}{5}E_1 = \frac{4}{5}\frac{\hbar\omega}{2} + \frac{1}{5}\frac{3\hbar\omega}{2} = \frac{7}{10}\hbar\omega$$

6.

$$\langle x \rangle = \int \left[\frac{2}{\sqrt{5}}\Phi_0 e^{i\frac{\omega}{2}t} + \frac{i}{\sqrt{5}}\Phi_1 e^{i\frac{3\omega}{2}t}\right] x \left[\frac{2}{\sqrt{5}}\Phi_0 e^{-i\frac{\omega}{2}t} - \frac{i}{\sqrt{5}}\Phi_1 e^{-i\frac{3\omega}{2}t}\right] dx =$$

only cross terms survive

$$-\frac{2i}{5} e^{-i\omega t}\int \Phi_0 x \Phi_1 \, dx + \frac{2i}{5} e^{i\omega t}\int \Phi_1 x \Phi_0 \, dx =$$

$$= -\frac{4}{5}\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\sqrt{\frac{1}{2\alpha}} = -\frac{4}{5}\frac{1}{\sqrt{2\alpha}}\sin\omega t$$