1. a) $\langle f(x) | S_a g(x) \rangle = \sqrt{a} \int f^*(x)\, g(ax)\, dx$

$\langle S_a f(x) | g(x) \rangle = \sqrt{a} \int f^*(ax)\, g(x)\, dx$

Obviously, in general $\langle f | S_a g \rangle \neq \langle S_a f | g \rangle$

$\langle f | S_a g \rangle \neq -\langle S_a f | g \rangle$

Hence, S_a is nonhermitian

b) $(\hat{x}\hat{p})^+ = \hat{p}^+ x^+ = \hat{p}\hat{x} \neq \hat{x}\hat{p}$ also $\hat{p}\hat{x} \neq -\hat{x}\hat{p}$

Therefore, $\hat{x}\hat{p}$ is nonhermitian

c) $\left(i(\hat{A}\hat{B} - \hat{B}\hat{A})\right)^+ = (\hat{A}\hat{B} - \hat{B}\hat{A})^+(-i) = (B^+A^+ - A^+B^+)(-i) =$

$= (BA - AB)(-i) = i(AB - BA)$ Thus, $i(\hat{A}\hat{B} - \hat{B}\hat{A})$ is hermitian

d) $(\hat{c} - \hat{c}^+)^+ = \hat{c}^+ - \hat{c} = -(\hat{c} - \hat{c}^+)$ Hence $\hat{c} - \hat{c}^+$ is antihermitian

e) $\left(\alpha\hat{x} - \beta\frac{d}{dx}\right)^+ = (\alpha\hat{x})^+ - \left(\beta\frac{d}{dx}\right)^+ = \alpha x + \beta\frac{d}{dx}$

This operator is nonhermitian

2. a) $\hat{P}(c_1\psi_1 + c_2\psi_2) = \langle \phi | c_1\psi_1 + c_2\psi_2 \rangle \phi = c_1\langle \phi | \psi_1 \rangle \phi + c_2 \langle \phi | \psi_2 \rangle \phi =$

$= c_1 \hat{P}\psi_1 + c_2 \hat{P}\psi_2 \implies \hat{P}$ is linear

b) $\langle f | \hat{P} g \rangle = \langle f | \underbrace{\langle \phi | g \rangle}_{\text{number}} \phi \rangle = \langle \phi | g \rangle \langle f | \phi \rangle$

$\langle \hat{P} f | g \rangle = \langle \langle \phi | f \rangle \phi | g \rangle = \langle \phi | f \rangle^* \langle \phi | g \rangle = \langle f | g \rangle \langle \phi | g \rangle$

We have shown that $\langle f | \hat{P} g \rangle = \langle \hat{P} f | g \rangle$

Hence $\hat{P} = \hat{P}^+$ and $\hat{P}$ is hermitian