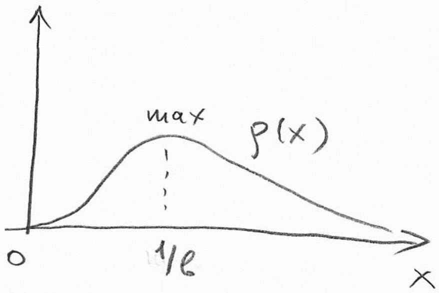
1) The probability distribution function is

$$\rho(x) = |\psi(x)|^2 = |A|^2 x^2 e^{-2bx}$$



The most probable position of the particle corresponds to the maximum of $\rho(x)$:

$$\frac{d\rho}{dx} = |A|^2 (2x - 2bx^2) e^{-2bx} = 0 \quad \Rightarrow \quad x_{max} = \frac{1}{b}$$

2) First let us normalize the wave function:

$$1 = \int_0^\infty |\psi(x)|^2 dx = |A|^2 \int_0^\infty x^2 e^{-2bx} dx = |A|^2 \frac{2!}{(2b)^3} \quad \Rightarrow |A|^2 = 4b^3$$

Here we used the value of the integral:

$$\int_0^\infty x^k e^{-\alpha x} dx = \frac{k!}{\alpha^{k+1}} \qquad k = 0, 1, 2, \dots \qquad \alpha > 0$$

The expected value of the momentum is

$$\langle p \rangle = \int_0^\infty \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi \, dx = -i\hbar |A|^2 \int_0^\infty x^2 e^{-bx - ikx} \left(1 + (ik - b)x\right) e^{-bx + ikx} dx$$

$$= -i\hbar |A|^2 \underbrace{\int_0^\infty (x - bx^2) e^{-2bx} dx}_{0} + \hbar |A|^2 k \underbrace{\int_0^\infty x^2 e^{-2bx} dx}_{\frac{1}{4b^3}} =$$

$$= \hbar \, 4b^3 \cdot k \cdot \frac{1}{4b^3} = \hbar k$$