

$$1) \int |\Psi(x,0)|^2 dx = 1$$

Given the mutual orthogonality of ψ_i 's we find that

$$1 = |A|^2 (1 + 4 + 1) \Rightarrow A = \frac{1}{\sqrt{6}}$$

and

$$\Psi(x,0) = \frac{1}{\sqrt{6}} \psi_1(x) - \frac{2}{\sqrt{6}} \psi_2(x) + \frac{i}{\sqrt{6}} \psi_3(x)$$

2) The possible outcomes and the corresponding probabilities are:

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2} \quad P_1 = \frac{1}{6}$$

$$E_2 = \frac{4\pi^2 \hbar^2}{2ma^2} = 4E_1 \quad P_2 = \frac{2}{3}$$

$$E_3 = \frac{9\pi^2 \hbar^2}{2ma^2} = 9E_1 \quad P_3 = \frac{1}{6}$$

$$3) \langle E \rangle = \sum_i P_i E_i = \frac{1}{6} E_1 + \frac{2}{3} \cdot 4E_1 + \frac{1}{6} \cdot 9E_1 = \frac{13}{3} E_1 = \frac{13\pi^2 \hbar^2}{6ma^2}$$

$$4) \Psi(x,t) = \frac{1}{\sqrt{6}} \psi_1(x) e^{-\frac{iE_1 t}{\hbar}} - \frac{2}{\sqrt{6}} \psi_2(x) e^{-\frac{iE_2 t}{\hbar}} + \frac{i}{\sqrt{6}} \psi_3(x) e^{-\frac{iE_3 t}{\hbar}}$$