

1) There are three states with  $l=1$ :  $m=+1, 0, -1$

2) See lecture 18 :

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$3) \quad L_x^2 = \frac{\hbar^2}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$L_y^2 = \frac{\hbar^2}{2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$L_x^4 = \frac{\hbar^4}{4} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

$$L_y^4 = \frac{\hbar^4}{4} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 2 \end{pmatrix}$$

$$\text{So } L_x^4 + L_y^4 = \hbar^4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The possible outcomes of  $L_x^4 + L_y^4$  measurements are its eigenvalues:  $\hbar^4$  (doubly degenerate) and  $2\hbar^4$

4) If  $|m_{\max}\rangle = |1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  then

$$\langle m_{\max} | L_x^4 + L_y^4 | m_{\max} \rangle = (1 \ 0 \ 0) \hbar^4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \hbar^4$$