The total wave function can be expanded in terms of partial waves:

$$\psi(r,\theta) = A\left[e^{ikz} + k\sum_{\ell=0}^{\infty} i^{\ell+1}(2\ell+1)a_\ell h_\ell^{(1)}(kr)\, P_\ell(\cos\theta)\right]$$

or given the Rayleigh formula for the plane wave

$$\psi(r,\theta) = A\sum_{\ell=0}^{\infty}\left[j_\ell(kr) + k i a_\ell h_\ell^{(1)}(kr)\right] i^\ell P_\ell(\cos\theta)$$

Since $\psi(a,\theta) = 0$ for any θ $\Longrightarrow$ $j_\ell(ka) + i k a_\ell h_\ell^{(1)}(ka) = 0$

Thus, $\quad a_\ell = \dfrac{i\, j_\ell(ka)}{k h_\ell^{(1)}(ka)}$

The total cross section in terms of the partial wave amplitudes is

$$\sigma_{tot} = 4\pi \sum_{\ell=0}^{\infty}(2\ell+1)|a_\ell|^2$$

In our case this becomes

$$\sigma_{tot} = \frac{4\pi}{k^2}\sum_{k=0}^{\infty}(2\ell+1)\left[\frac{j_\ell(ka)}{h_\ell^{(1)}(ka)}\right]^2$$

In the low energy limit $ka \ll 1$. Using the fact that

$$j_\ell(x) \to \frac{2^\ell \ell!}{(2\ell+1)!}x^\ell \qquad h_\ell \to -\frac{(2\ell)!}{2^\ell \ell!}x^{-\ell-1}$$

$$\frac{j_\ell(x)}{h_\ell^{(1)}(x)} = \frac{j_\ell(x)}{j_\ell(x) + i n_\ell(x)} \overset{x\to 0}{\approx} \frac{j_\ell(x)}{n_\ell(x)} \approx \frac{2^\ell \ell!}{(2\ell+1)!}x^{2\ell+1}$$

In the low energy limit we can restrict ourselves with s-wave scattering only ($\ell=0$). With that

$$\sigma_{tot} = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\frac{2^\ell \ell!}{(2\ell+1)!}ka^{2\ell+1}\right]^2 \approx \frac{4\pi}{k^2}k^2a^2 = 4\pi a^2$$

In the context of this problem "low" energy means

$$ka = \frac{\sqrt{2mE}\,a}{\hbar} \ll 1 \qquad \text{or} \qquad E \ll \frac{\hbar^2}{2ma^2}$$