

No, $(\alpha + \beta x^2)e^{-\gamma x^2}$ is a very bad choice for approximating the wave function of the first excited state. Since the potential $V(x) = A x^2 \frac{a^2 - x^2}{b^2 - x^2}$ is an even function, the exact eigenstates of the Hamiltonian are either even or odd. The ground state is even, the first excited state is odd, the second excited state is even, and so on. Hence, function $(\alpha + \beta x^2)e^{-\gamma x^2}$, which is even, will always be completely orthogonal to the exact wave function of the first excited state regardless of any particular choice of α , β , and γ .

A good trial wave function for approximating the first excited state would be $x e^{-\gamma x^2}$.