Here we are dealing with a degenerate level $E_1^{(0)} = E_2^{(0)} = \alpha$. Hence, before we can apply the perturbation theory formulae we have to find the proper zero-order basis, in which H' is diagonal.

$$H' = \beta \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad H' \vec{c}_i = \varepsilon \vec{c}_i \qquad \begin{vmatrix} \beta - \varepsilon & \beta \\ \beta & \beta - \varepsilon \end{vmatrix} = (\beta - \varepsilon)^2 - \beta^2 = 0$$

$$\beta - \varepsilon = \pm \beta \qquad \varepsilon_{1,2} = 0, 2\beta$$

The eigenvector corresponding to $\varepsilon_1 = 0$:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} c_{11} \\ c_{12} \end{pmatrix} = 0 \qquad c_{11} = -c_{12} \qquad \vec{c}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The eigenvector corresponding to $\varepsilon_2 = 2\beta$:

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_{21} \\ c_{22} \end{pmatrix} = 0 \qquad c_{21} = c_{22} \qquad \vec{c}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

So the proper basis in which H' becomes diagonal is

$$\vec{c}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \vec{c}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and H' in it looks as follows: $H' = \begin{pmatrix} 0 & 0 \\ 0 & 2\beta \end{pmatrix}$

Now we can compute the first order corrections to the energy levels:

$$E_1^{(1)} = \vec{c}_1^{\,T} \cdot H' \cdot \vec{c}_1 = \frac{1}{\sqrt{2}} (1 \ -1) \, \beta \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

$$E_2^{(1)} = \vec{c}_2^{\,T} H' \vec{c}_2 = \frac{1}{\sqrt{2}} (1 \ 1) \, \beta \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2\beta$$

Second order corrections:

$$E_1^{(2)} = 0$$

$$E_2^{(2)} = 0$$

$\leftarrow$ this is because we only have two states and all off-diagonal matrix elements are $H'_{mn} = 0 \quad m \neq n$