1. H'_{ij} must be small compared to the distance between the (non-degenerate) energy levels. Since the units of the delta-function are $\frac{1}{x}$ and the natural length of the problem is a, we can estimate

$$\gamma \frac{1}{a}\frac{1}{a} << \frac{\hbar^2}{ma^2} \quad \text{or} \quad \gamma << \frac{\hbar^2}{m}$$

2. The third excited energy level is two-fold degenerate:

$n_x = 1 \quad n_y = 3 \qquad \psi_1 = \frac{2}{a}\sin\frac{\pi x}{a}\sin\frac{3\pi y}{a}$

$n_x = 3 \quad n_y = 1 \qquad \psi_2 = \frac{2}{a}\sin\frac{3\pi x}{a}\sin\frac{\pi y}{a}$

So we need to apply the perturbation theory for the degenerate states

$$H'_{11} = \frac{4\gamma}{a^2}\underbrace{\int_0^a \sin\frac{\pi x}{a}\,\delta(x-\tfrac{a}{2})\sin\frac{\pi x}{a}\,dx}_{1}\ \underbrace{\int_0^a \sin\frac{3\pi y}{a}\,\delta(y-\tfrac{a}{2})\sin\frac{3\pi y}{a}\,dy}_{1} = \frac{4\gamma}{a^2}$$

$$H'_{22} = H'_{11} = \frac{4\gamma}{a^2} \quad (\text{symmetry } x \rightleftarrows y)$$

$$H'_{12} = \frac{4\gamma}{a^2}\underbrace{\int_0^a \sin\frac{\pi x}{a}\,\delta(x-\tfrac{a}{2})\sin\frac{3\pi x}{a}\,dx}_{-1}\ \underbrace{\int_0^a \sin\frac{3\pi y}{a}\,\delta(y-\tfrac{a}{2})\sin\frac{\pi y}{a}\,dy}_{-1} = \frac{4\gamma}{a^2}$$

$$H' = \frac{4\gamma}{a^2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \qquad \begin{vmatrix}1-\varepsilon & 1\\ 1 & 1-\varepsilon\end{vmatrix} = 0 \qquad (1-\varepsilon)^2 = 1,\quad \varepsilon = 0, 2$$

So in the first order we get

$$E^{(1)} = 0, \frac{8\gamma}{a^2}$$

The degeneracy is lifted and the energy level of the third excited state splits into two. The distance between the latter two is $\Delta E = \frac{8\gamma}{a^2}$

In the case of the second excited state there is no degeneracy:

$n_x = 2 \quad n_y = 2 \qquad \psi = \frac{2}{a} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{a}$

Moreover, all matrix elements of H' are zeros

$$H'_{11} = H'_{22} = H'_{12} = 0$$

Hence, nothing happens to this energy level.