

If we look at the exact wave functions of the hydrogen atom for $n=2$ and $\ell=1$, namely

$$\Psi_{n\ell m} = R_{n\ell}(r) Y_e^m(\theta, \varphi)$$

$$R_{21} = \frac{1}{\sqrt{24}} \frac{1}{a^3} \frac{1}{a} e^{-\frac{r}{2a}} \quad Y_1^{\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \frac{x \pm iy}{r} \quad Y_1^0 = \sqrt{\frac{3}{4\pi}} \frac{z}{r}$$

we immediately realize that Ψ has the same functional form as $R_{21}(Y_1^{+1} - Y_1^{-1})$ — a function corresponding to a definite value of n . So obviously with the optimal values of $\alpha=0$ and $\beta=\frac{1}{2a}$ we can reproduce the exact energy of the first excited state ($n=2$):

$$E = -\frac{1}{2n^2} \text{ hartree} = -\frac{1}{8} \text{ hartree}$$

Hence Ψ is an extremely good choice (upon proper choice of α and β it yields the exact solution)