1. The value of γ is irrelevant (as long as it is not equal to $-\infty$) because $V(x) = \infty$ outside of interval $0 \le x \le a$ and so H' will remain infinite there.

The applicability of the perturbation theory is determined by the criterion

$$|H'_{nm}| \ll |E_n^{(0)} - E_m^{(0)}|$$

In our case $H'_{nm} = \langle \psi_n^{(0)} | \beta | \psi_m^{(0)} \rangle = \beta \delta_{nm} = 0 \quad n \neq m$

So the perturbation theory should be applicable for any value of β

2. $E_n^{(1)} = \langle \psi_n^{(0)} | \beta | \psi_n^{(0)} \rangle = \beta$

i.e. all energy levels are shifted by a constant β

3. $E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{nm}|^2}{E_n^{(0)} - E_m^{(0)}} = 0$

This is because $H'_{nm} = 0 \quad n \neq m$

4. Yes the above results make perfect sense. We know that the energy levels are shifted by β when we solve the problem exactly (it is still a particle in a box, just with a different reference point for the energy). All higher order corrections (second and higher) vanish because $H'_{nm} = 0 \quad n \neq m$