The eigenvalues and eigenstates of the unperturbed Hamiltonian

$$H = \begin{pmatrix} 2 & i \\ -i & 1 \end{pmatrix}$$

are :

$$E_1^{(0)} = 1 \qquad \psi_1^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \qquad E_2^{(0)} = 3 \qquad \psi_2^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

The first order corrections to the energies are :

$$E_1^{(1)} = \langle \psi_1^{(0)} | V | \psi_1^{(0)} \rangle = \frac{1}{2}(1 \;\; i)\,\gamma \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \gamma$$

$$E_2^{(1)} = \langle \psi_2^{(0)} | V | \psi_2^{(0)} \rangle = \frac{1}{2}(1 \; -i)\,\gamma \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \gamma$$

The second order corrections are

$$E_1^{(2)} = \sum_{\substack{m=1 \\ m \neq 1}}^{2} \frac{|V_{m1}|^2}{E_1^{(0)} - E_m^{(0)}} = \frac{|V_{21}|^2}{1-3} = -\frac{|V_{21}|^2}{2}$$

$$E_2^{(2)} = \sum_{\substack{m=1 \\ m \neq 2}}^{2} \frac{|V_{m2}|^2}{E_2^{(0)} - E_m^{(0)}} = \frac{|V_{12}|^2}{2}$$

Now $V_{12} = V_{21}^{*} = \langle \psi_1^{(0)} | V | \psi_2^{(0)} \rangle = \frac{1}{2}(1 \;\; i)\,\gamma \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = 2i\gamma$

So

$$E_1^{(2)} = -2\gamma^2 \qquad E_2^{(2)} = 2\gamma^2$$

In the end we have

$$E_1 = 1 + \gamma - 2\gamma^2 + \ldots \qquad E_2 = 3 + \gamma + 2\gamma^2 + \ldots$$