The zero-order wave functions for a particle in 2D box are products of 1D box solutions, namely

$$\Psi_N^{(0)}(x,y) = \psi_n(x)\,\psi_k(y) = \frac{2}{a}\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right) \quad \text{where } n,k = 1,2,3,\ldots$$

1) The applicability of the perturbation theory implies

$$\langle \Psi_N^{(0)} | V | \Psi_N^{(0)} \rangle \ll E_N^{(0)} - E_M^{(0)} \sim \frac{\hbar^2}{ma^2}$$

Now

$$\langle \Psi_N^{(0)} | \beta\,\delta(x-\tfrac{a}{4})\,\delta(y-\tfrac{a}{4}) | \Psi_N^{(0)} \rangle \sim \beta\frac{1}{a^2}$$

Therefore we get

$$\beta \ll \frac{\hbar^2}{m}$$

2) The first excited state is two-fold degenerate:

$n=1 \quad k=2 \qquad \Psi_1^{(0)}(x,y) = \psi_1(x)\,\psi_2(y)$

$n=2 \quad k=1 \qquad \Psi_2^{(0)}(x,y) = \psi_2(x)\,\psi_1(y)$

so we must use the degenerate perturbation theory:

$$V_{11} = \langle \Psi_1(x,y) | V | \Psi_1(x,y) \rangle = \beta\,\underbrace{\langle \psi_1(x) | \delta(x-\tfrac{a}{4}) | \psi_1(x) \rangle}_{I_{11}}\,\underbrace{\langle \psi_2(y) | \delta(y-\tfrac{a}{4}) | \psi_2(y) \rangle}_{I_{22}}$$

Similarly, $V_{12} = \beta I_{12} I_{21}$ and $V_{22} = \beta I_{22} I_{11}$

Now, $I_{11} = \frac{2}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)\delta(x-\tfrac{a}{4})\sin\left(\frac{\pi x}{a}\right)dx = \frac{2}{a}\left(\sin\frac{\pi}{4}\right)^2 = \frac{2}{a}\cdot\frac{1}{2} = \frac{1}{a}$

$I_{12} = I_{21} = \frac{2}{a}\int_0^a \sin\left(\frac{2\pi x}{a}\right)\delta(x-\tfrac{a}{4})\sin\left(\frac{\pi x}{a}\right)dx = \frac{2}{a}\sin\frac{\pi}{2}\sin\frac{\pi}{4} = \frac{\sqrt{2}}{a}$

$I_{22} = \frac{2}{a}\int_0^a \sin\left(\frac{2\pi x}{a}\right)\delta(x-\tfrac{a}{4})\sin\left(\frac{2\pi x}{a}\right)dx = \frac{2}{a}\left(\sin\frac{\pi}{2}\right)^2 = \frac{2}{a}$

Therefore, $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} \beta I_{11} I_{22} & \beta I_{12}^2 \\ \beta I_{12}^2 & \beta I_{22} I_{11} \end{pmatrix} = \frac{2\beta}{a^2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

The eigenvalues of this matrix give the first-order corrections to the energy:

$$E^{(1)} = 0 \quad \text{and} \quad E^{(1)} = \frac{4\beta}{a^2}$$