1 Let us expand the trial wave function ϕ in terms of the eigenfunctions of the Hamiltonian, ψ_i, which correspond to eigenvalues E_i ($E_1 < E_2 < E_3 \ldots$). We can do that because ψ_i's form a complete basis.

$$\phi = \sum_{i=1}^{\infty} c_i \psi_i \qquad \langle \psi_i | \psi_j \rangle = \delta_{ij}$$

The fact that ϕ is orthogonal to ψ_1 means that C_1 is zero:

$$0 = \langle \psi_1 | \phi \rangle = \langle \psi_1 | \sum_i c_i \psi_i \rangle = \sum_i c_i \underbrace{\langle \psi_1 | \psi_i \rangle}_{\delta_{1i}} = C_1$$

Then

$$\langle \phi | \phi \rangle = \langle \sum_i c_i \psi_i | \sum_j c_j \psi_j \rangle = \sum_{ij} c_i^* c_j \langle \psi_i | \psi_j \rangle = \sum_i |c_i|^2$$

$$\langle \phi | H | \phi \rangle = \langle \sum_i c_i \psi_i | H | \sum_j c_j \psi_j \rangle = \sum_{ij} c_i^* c_j \langle \psi_i | H | \psi_j \rangle = \sum_{ij} c_i^* c_j E_j \langle \psi_i | \psi_j \rangle = \sum_i |c_i|^2 E_i$$

$$E_{trial} = \frac{\langle \phi | H | \phi \rangle}{\langle \phi | \phi \rangle} \quad \text{or} \quad E_{trial} \langle \phi | \phi \rangle = \langle \phi | H | \phi \rangle$$

With that we get

$$E_{trial} \sum_{i=2}^{\infty} |c_i|^2 = \sum_{i=2}^{\infty} |c_i|^2 E_i \geq \sum_{i=2}^{\infty} |c_i|^2 E_2 = E_2 \sum_{i=2}^{\infty} |c_i|^2$$

Thus

$$E_{trial} \geq E_2$$

2 The functions that do not have $e^{-\alpha x^2}$ of $e^{-\alpha x^4}$ are not square integrable and so are not meaningful for approximating any bound state. The exact ground state wave function must be even because $V(x)$ is even. Therefore, only those functions that contain x^1 will be orthogonal to it. That leaves us two choices that are suitable for approximating the first excited state: $xe^{-\alpha x^2}$ and $xe^{-\alpha x^4}$. We know that for potentials that go to zero (or constant) at $x = \pm\infty$ the wave function must decay exponentially (i.e. $e^{-\gamma |x|}$). Apparently $e^{-\alpha x^2}$ decays slower than $e^{-\alpha x^4}$ and is likely to be a better choice. So the answer is (g) $Axe^{-\alpha x^2}$