1. $E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle = \int_0^a |\psi_n^{(0)}(x)|^2 H'(x)\, dx =$

$= A \int_0^{a/2} |\psi_n^{(0)}(x)|^2 dx$, where $\psi_n^{(0)}(x)$ is the wave function of the particle in box $0 \le x \le a$

$|\psi_n^{(0)}(x)|^2$ is symmetric/even with respect to the center of the box (point $x = \frac{a}{2}$). So we can write

$$\int_0^{a/2} |\psi_n^{(0)}(x)|^2 dx = \frac{1}{2} \underbrace{\int_0^a |\psi_n^{(0)}(x)|^2 dx}_{\psi_n^{(0)} \text{ is normalized}} = \frac{1}{2}$$

Therefore

$$E_n^{(1)} = \frac{A}{2}$$

2. The applicability of the perturbation theory is determined from the relation

$$|H'_{nm}| \ll |E_n^{(0)} - E_m^{(0)}|$$

In our case we have $H'_{nm} = \int_0^a \psi_n^{(0)}(x) H'(x) \psi_m^{(0)}(x)\, dx \sim A$

while $E_n^{(0)} - E_m^{(0)} \sim \frac{\hbar^2}{ma^2}$ (recall that $E_n^{(0)} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$).

So if

$$A \ll \frac{\hbar^2}{ma^2}$$

then the perturbation theory is applicable