a) The unperturbed Hamiltonian is:

$$H^{\circ} = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{S}\cdot\vec{B} = -\gamma S_y B_y = -\frac{\gamma\hbar B}{2}\sigma_y = -\frac{\gamma\hbar B}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

b) The eigenvalues and eigenfunctions of H° are:

$$E_1^{(0)} = \frac{\gamma\hbar B}{2} \qquad \psi_1^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$E_2^{(0)} = -\frac{\gamma\hbar B}{2} \qquad \psi_2^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

c) The perturbing Hamiltonian H' is

$$H' = -\vec{\mu}\cdot\vec{B}' = -\frac{\gamma\hbar\alpha B}{2}\sigma_z = -\frac{\gamma\hbar\alpha B}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The first order corrections to the energy levels are:

$$E_1^{(1)} = \langle\psi_1^{(0)}|H'|\psi_1^{(0)}\rangle = -\frac{\gamma\hbar\alpha B}{4}\begin{pmatrix} 1 & i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = 0$$

$$E_2^{(1)} = \langle\psi_2^{(0)}|H'|\psi_2^{(0)}\rangle = -\frac{\gamma\hbar\alpha B}{4}\begin{pmatrix} 1 & -i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = 0$$

vanish

The second order corrections are evaluated as follows:

$$H'_{12} = H'^{*}_{21} = \langle\psi_1^{(0)}|H'|\psi_2^{(0)}\rangle = -\frac{\gamma\hbar\alpha B}{4}\begin{pmatrix} 1 & i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = -\frac{\gamma\hbar\alpha B}{2}$$

$$E_1^{(2)} = \sum_{m\neq 1}^{2}\frac{|H'_{m1}|^2}{E_1^{(0)}-E_m^{(0)}} = \frac{\gamma^2\hbar^2\alpha^2B^2/4}{\frac{\gamma\hbar B}{2}+\frac{\gamma\hbar B}{2}} = \frac{\gamma\hbar B}{4}\alpha^2$$

$$E_2^{(0)} = \sum_{m\neq 2}^{2}\frac{|H'_{m1}|^2}{E_2^{(0)}-E_m^{(0)}} = \frac{\gamma^2\hbar^2\alpha^2B^2/4}{-\frac{\gamma\hbar B}{2}-\frac{\gamma\hbar B}{2}} = -\frac{\gamma\hbar B}{4}\alpha^2$$