We can use the first order time dependent perturbation theory to find the coefficient $c_2(T)$ in the expansion

$$\Psi(x,t) = \sum_n' c_n(t)\psi_n(x)e^{-\frac{iE_n t}{\hbar}}$$

The first order coefficient is

$$c_2^{(1)}(t) = \frac{1}{i\hbar}\int_0^t H'_{12}\, e^{i\omega_{21}t'}\,dt'$$

where $H'_{12} = \langle \psi_1|H'|\psi_2\rangle$ and $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

With that we obtain

$$H'_{12} = \frac{2V_0}{a}\int_0^{a/2}\sin\frac{\pi x}{a}\sin\frac{2\pi x}{a}\,dx = \frac{V_0}{a}\int_0^{a/2}\left[\cos\frac{\pi x}{a} - \cos\frac{3\pi x}{a}\right]dx$$

$$= \frac{V_0}{a}\left[\frac{a}{\pi}\sin\frac{\pi x}{a}\Big|_0^{a/2} - \frac{a}{3\pi}\sin\frac{3\pi x}{a}\Big|_0^{a/2}\right] = \frac{V_0}{a}\left[\frac{a}{\pi} + \frac{a}{3\pi}\right] = \frac{4V_0}{3\pi}$$

$$c_2(t) \approx c_2^{(1)}(t) = \frac{1}{i\hbar}\cdot\frac{4V_0}{3\pi}\int_0^t e^{i\omega_{21}t'}\,dt' = -\frac{4V_0}{3\pi\hbar\omega_{21}}\left(e^{i\omega_{21}t} - 1\right) =$$

$$= -\frac{8iV_0}{3\pi\hbar\omega_{21}}e^{\frac{i\omega_{21}t}{2}}\sin\frac{\omega_{21}t}{2} \qquad \omega_{21} = (2^2-1^2)\frac{\hbar^2\pi^2}{2ma^2\hbar} = \frac{3\pi^2\hbar}{2ma^2}$$

$$P_{1\to 2}(T) = |c_2(T)|^2 = \left[\frac{16}{9\pi^3}\frac{V_0 m a^2}{\hbar^2}\right]^2\sin^2\frac{3\pi^2\hbar T}{4ma^2}$$