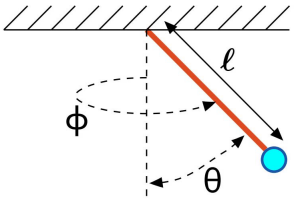
PHYS 505: Classical Mechanics (graduate) - Fall 2015
Homework #2, due Friday October 2, in class

Motion of a variable-mass system. The Lagrange method.

1. A spherical raindrop falls from a height y_0 to the Earth's surface. As it falls, the raindrop grows in mass through the absorption of moisture from its surrounding. The growth rate is proportional to the instantaneous surface area of the raindrop. Assuming that the raindrops starts at rest with a radius r_0, calculate how long it will take to fall to the Earths surface. Neglect air friction.

2. (This problem was given at the diagnostic test) A point mass m is attached to the ceiling by a massless rod of length l. The position of this point mass is then determined by two angles, ϕ and θ.



   (a) Write down the expressions for the kinetic and potential energies in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$.

   (b) Using the Lagrangian method, derive equations of motion for both ϕ and θ?

   (c) Find an expression for the angular momentum L_z in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$.

   (d) Rewrite the equations of motion for θ so that any mention of $\dot{\phi}$ is replaced by L_z.

   (e) Consider a circular trajectory of constant $\theta = \pi/4$. What is L_z and what is the frequency of small oscillations of θ about $\pi/4$? Give answers in terms of m, g, l.

3. A particle of mass m moves without friction on a cycloid. The cycloid (you can look it up in Wikipedia) is given by $x = R(\xi - \sin\xi)$ and $y = R(1 + \cos\xi)$, where $0 \leq \xi \leq 2\pi$ and R is a constant.

   (a) Determine the Lagrangian of the system

   (b) Find the equation of motion

   (c) Solve the equation of motion. *Hint: to solve the equation of motion it might be convenient to make a substitution* $u = \cos(\xi/2)$

4. Problem 1.22 in Goldstein.

5. Problem 1.23 in Goldstein.