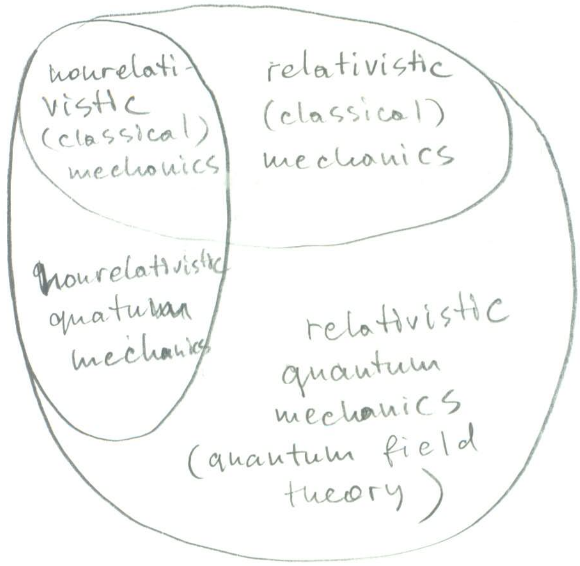
Introductory notes

Mechanics is a branch of natural science that studies the behavior of material bodies that are subjected to displacements and forces

Force is a form of interaction that, when unopposed, changes the motion (or a state) of a material body

The term classical mechanics is used primarily to distinguish it from quantum mechanics - a theory that concerns microscopic objects such as atoms. Classical mechanics aims to describe the motion of macroscopic bodies (including astronomical objects). Classical mechanics, as any physical theory, has a certain domain of applicability.

One can divide classical mechanics into the nonrelativistic an relativistic one. The relativistic classical mechanics concerns the motion of objects whose velocities are comparable to the speed of light. Nonrelativistic classical mechanics is a subdomain of the relativistic classical mechanics. A widely used term for nonrelativistic classical mechanics is Newtonian mechanics — a particular form based on physical concepts and mathematical methods invented by Isaac Newton and other scientists. There are more abstract and general formulations of nonrelativistic classical mechanis called Lagrange, Hamiltonian, and Hamilton-Jacobi mechanics. We will consider those in this course



The Lagrangian and Hamiltonian formulations of mechanics (also often referred to as Analytical mechanics) make an emphasis on system's energy rather than on forces.

There are several important concepts that are involved into the foundation of mechanics: mass, time, space, force (or interaction). While deep understanding what those concepts are may overlap with the subject of philosophy, we have a good intuitive understanding of those concepts (sufficient for nonrelativistic classical mechanics at least). For example, mass is a property that describes how inert or resistant a free object is to a change in its mechanical state.

A very important concept in mechanics is that of a particle. By this we mean an object whose size may be neglected in describing its motion. It, of course, depends on the system we are studying. For example, the Earth can often be considered as particle when we study its motion around the Sun. However, the same object (the Earth) cannot be regarded as particle when we study its rotation about its own axis.

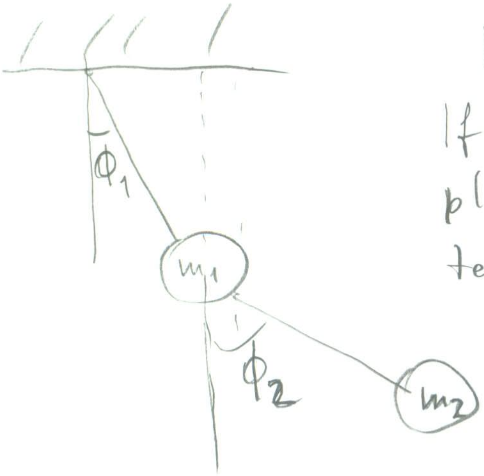
The laws of mechanics are such (and this has been veryfied empiracally) that a state of a particle is completely defined by its position and velocity. This comes from the fact that the equation that describes the motion of a particle (the second Newton's law) is a second order differential equation: In one dimension

$$m\ddot{x} = F$$

Given $F(x,t)$ and $x(0)$ and $V(0) = \left.\frac{dx}{dt}\right|_{t=0}$ we can determine the position of the particle at any instant.

Often we use the usual Cartesian coordinates to describe the positions of particles. To define the position of a system of N particles in 3D space it is necessary to specify N radius vectors or $3N$ coordinates. What may happen, however, is that sometimes it is more convenient or natural for some particular system to use non-Cartesian coordinates. Also, in a mechanical system we may have constraints. For example, a bead on a ring is constrained to move on that ring only and its position is most naturally defined by an angle.

The number of independent quantities, which must be specified in order to unambiguously define the position of a system is called the number of degrees of freedom.

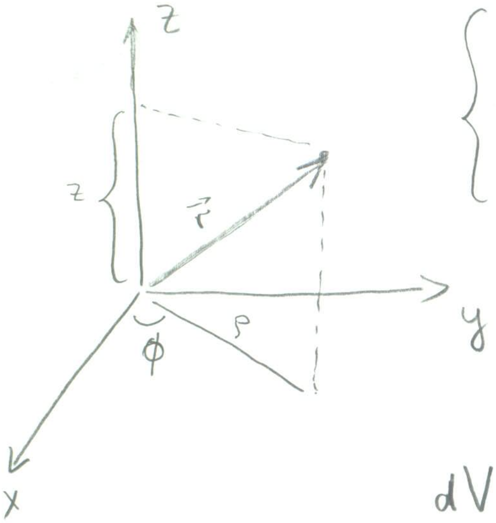


Example: double pendulum

If the motion is constrained in a plane then the position of the system is unambiguously specified by two angles, ϕ_1 and ϕ_2.

However if the motion of the double pendulum is not constrained in a plane then we need four angles. Still this number is less than 6 — the number of coordinates necessary to specify the position of two free particles.

If we consider the position of a single particle it is sometimes convenient to use cylindrical or spherical coordinates

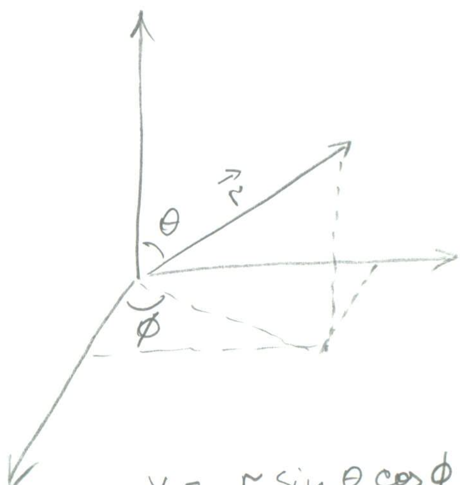


$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

$$\begin{cases} \rho = \sqrt{x^2+y^2} \\ \phi = \arctan\left(\frac{y}{x}\right) \\ z = z \end{cases}$$

$$dV = \rho\, d\rho\, d\phi\, dz$$

element of volume



$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\phi = \arctan\left(\frac{y}{x}\right)$$

$$dV = r^2\sin\theta\, d\phi\, d\theta\, dr$$

The gradient and Laplacian (∇^2) in the cylindrical coordinates is

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho A_\rho + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

In the spherical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\sin\theta A_\theta + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\sin\theta\frac{\partial f}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$